2024/FYUG/ODD/SEM/
STADSC–201T/149

FYUG Odd Semester Exam., 2024

STATISTICS

(3rd Semester)

Course No. : STADSC–201T

(Algebra)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Define diagonal matrix and skew-symmetric matrix.

(b) Prove that a square matrix is a symmetric matrix, if and only if A and A' are equal.

(c) Define triangular matrix and involutory matrix.

2. Answer *either* (a) *or* (b) : 10

(a) (i) Define idempotent matrix. Show that every square matrix is uniquely expressible as a sum of symmetric matrix and a skew-symmetric matrix. 1+3=4

(ii) Define orthogonal matrix. If

$$A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

then show that A is an orthogonal matrix. 3

(iii) Let A and B be two square matrices of order n and λ be a scalar. Then prove that $\operatorname{tr}(AB) = \operatorname{tr}(BA)$. 3

(b) (i) Show that every square matrix A can be uniquely expressed as $P + iQ$, where P and Q are Hermitian matrices. 6

(ii) Define inverse of a matrix. If A and B be two n-rowed non-singular matrices, then prove that $(AB)^{-1} = B^{-1}A^{-1}$. 4

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Define determinant of a square matrix.

(b) Define echelon form and normal form of a matrix.

(c) Define row rank and column rank of a matrix.

4. Answer *either* (a) *or* (b) : 10

(a) (i) Show that—

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$ 4

(ii) Prove that—

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$ 3

(iii) Prove that the necessary and sufficient condition for a square matrix A to possess inverse is that $|A| \neq 0$. 3

(b) (i) Find A^{-1} of the following matrix : 3

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

(ii) Define rank of a matrix. Show that rank of product AB of two matrices cannot be greater than the smaller of the ranks of A and B. 1+3=4

(iii) Prove that the rank of the transpose of a matrix is same as that of the original matrix. 3

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Define quadratic form over a field.

(b) Briefly explain characteristic matrix and characteristic equation of a matrix.

(c) Write down the matrix of the following quadratic form :

$$x_1^2 + 2x_2^2 - 5x_3^2 - x_1x_2 + 4x_2x_3 - 3x_3x_4$$

6. Answer *either (a) or (b)* : 10

(a) *(i)* State and prove Cayley-Hamilton theorem. 4

(ii) Find the characteristic roots of the matrix :

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and show that $AA' = I$, where A' is the transpose of A. 3

(iii) Prove that if the characteristic roots of A are $\lambda_1, \lambda_2, \cdots, \lambda_n$, then the characteristic roots of A^2 are $\lambda_1^2, \lambda_2^2, \cdots, \lambda_n^2$. 3

(b) *(i)* Check the consistency of the following equations :

$$\begin{aligned} x + 2y - z &= 3 \\ 3x - y + 2z &= 1 \\ 2x - 2y + 3z &= 2 \\ x - y + z &= -1 \end{aligned}$$

If they are consistent, solve them. 7

(ii) Prove that the system of equations $AX = B$ is consistent, i.e., it possesses a solution, if and only if the coefficient matrix A and the augmented matrix $[A : B]$ are of same rank. 3

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Define group with an example.

(b) Define basis of a vector space.

(c) Prove that if two vectors are linearly dependent, then one of them is a scalar multiple of the other.

8. Answer *either (a) or (b)* : 10

(a) *(i)* Show that the set Z of all integers forms an infinite Abelian group with respect to addition of integers. 3

(ii) Define ring. If $(R, +, \cdot)$ is a ring, then for all $a, b, c \in R$, prove that—

1. $a \cdot (b - c) = a \cdot b - a \cdot c$
2. $a \cdot (-b) = -(a \cdot b) = (-a) \cdot b$
3. $(b - c) \cdot a = b \cdot a - c \cdot a$ 1+3=4

(iii) Define field. Show that every field is an integral domain. 3

(b) *(i)* Define vector subspace. Show that the union of two subspaces is a subspace if and only if one is contained in the other. 1+3=4

(ii) Show that any subset of a linearly independent set of vectors $V(F)$ is linearly independent. 3

(iii) Show that the three vectors $\{(1, 2, 0), (0, 3, 1), (-1, 0, 1)\}$ of R^3 are linearly independent. 3

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Determine a polynomial whose roots are $-3, -1, 4, 5$.

(b) If the roots of the equation

$$x^3 + 3px^2 + 3qx + r = 0$$

are in AP, then prove that $2p^3 - 3pq + r = 0$.

(c) Find the roots of the equation $x^3 - 3x^2 + 4 = 0$, if two roots of it being equal.

10. Answer *either (a) or (b)* : 10

(a) *(i)* State and prove the fundamental theorem of algebra. 4

(ii) If the sum of two roots of $x^3 + px^2 + qx + r = 0$ is equal to the third root, then show that $p^3 - 4pq + 8r = 0$. 6

(b) *(i)* Solve the following by Cardan's method : 4

$$x^3 - 15x^2 - 33x + 847 = 0$$

(ii) Find the roots of the equation

$$x^3 + 7x^2 + 14x - 9 = 0$$

if the roots are in GP. 2½

(iii) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the values of $\Sigma\alpha^3\beta^3$, $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$ and $\Sigma(\alpha-\beta)^2$. 3½

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